A new integral criterion for parameter optimisation of dynamic systems with Evolution Strategy

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Abstract

This paper presents an optimisation criterion for parameter tuning of control systems, which is suitable for evolutionary optimisation i.e. it fulfils the demand of Strong Causality [1]. It deals with a modification of the known integral of time multiplied by the squared error criterion (ITSE). In order to influence a characteristic value of a signal it is unhandy to add a special term to the ITSE for increasing the selection pressure in Evolution Strategies. An evident possibility is to divide the integral criterion in special error sections for each characteristic value. The addition of integral sections in the time horizon is straight forward and has always the same units. For one characteristic value stands one part of the error signal. The differences in relation to the ITSE are given and a small non-linear example is discussed.

Introduction

The use of Evolutionary Algorithms (EA) for system identification and control in practice extends the range of solvable problems. Standard design methods and optimisation methods in control are mostly defined only for linear time invariant systems. For the wide range of non-linear systems only local or particular solutions exist. The main problem of deterministic methods is that there are mostly no model equations.

Evolutionary Algorithms especially Evolution Strategies (ES) are robust in the sense of noise and do not require an analytical model for optimisation. In order to increase the solution efficiency and to get a higher convergence speed it is necessary to adapt the problem description for the evolutionary method. In general an smooth fitness landscape leads to faster optimisation. This is the content of Evolution Technique, i.e. building of quality or fitness functions. This paper deals with an application of ES [1] in control system design and optimisation. But the approach is also applicable for other optimisation methods. The characteristic feature is not the actual optimisation task with EA [2], [3],[4] but the shaping and design of the quality function. In order to get an efficiency optimisation the principle of Strong Causality [1] is used

Recent developments in fields of parameter tuning demonstrate the use of ITSE criterions in control system design with ES. This approach has two main disadvantages. Firstly, the solutions are often not definite [5] and meet the requirements of the characteristic values often inaccurately. The aim is not to optimise the characteristic values but to minimise the quadratic error. Secondly, the solutions are often conservative because the correcting variable is minimised in relation to the signal error and it is not set to the given maximum in order to speed up the transient response.

The problem of correcting variable is often ignored and in theoretical developments without meaning. The addition of penalty terms to quality function is used to level up the first disadvantage [6]. A penalty function changes the selection pressure into the direction of satisfying a given constraint. The summation of the integral criterion and several penalty functions or terms leads to the problem of weighting and to the distortion of the fitness landscape. The active weighting in the fitness function requires an exact knowledge about dimensions and correlations between given constraints in the near and in the far of the optimum. Unlucky weightings can lead to distorting the relationship and to running in a local optimum. The possible occurrence of ridges in the fitness landscape reduces the convergence speed and can lead to the getting stuck of optimisation algorithm.

Universally valid values are problem dependent and therefore ill defined beforehand. The reaching of robust stability of the resulting dynamic system is always to demand and mandatory in praxis. In non-linear black-box systems it is difficult to guarantee a robust stability for the amount of manipulated values and mostly shown for a bounded range only. All the problems of stability of non-linear black-box systems cannot be treated in detail but should not be forgotten here.

The paper rather deals with a separating of the integral criterion concerning the simulation time, and whit the transfer of some characteristic values into the integral form. The following study bases on a given identification structure without a special treatment of stability.

Basics in performance indices and signal characteristics

The aim of control optimisation is to determine the control parameter in order to minimise the error signal *e(t)* as shown in Figure 1.

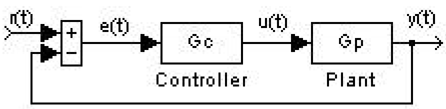

Figure 1: Closed-loop control system

The general form and an often used performance index is shown below.

$$I = \int_0^T f\left(r(t), e(t), u(t), y(t), t\right) dt \qquad , f = f_{ITSE}\left(e(t), u(t), t\right) = t^n \left(e^2(t) + a \cdot u^2(t)\right) \qquad , n > 1, a > 0 \tag{1}$$

For more practical points of view boundary conditions are set, e.g. limitation of correcting variable *u(t)* [7],[8]. A desired input signal can be given by a well-defined reference input, by a desired tube [5] or by characteristic values [6].

In general a combination of reference input and characteristic values for special requirements concerning the output signal is often used. With the use of characteristic values an indirect avoidance of signal ambiguity [5],[9] can be reached. The characteristic values result directly from the application and can be considered as standard. Other supplements are possible, e.g. pole placement, frequency change values or all quantities of other system restrictions. Standard performance measures are usually defined in terms of the unit step response and are described in [10].

- The swiftness of the response is defined by the rise time t_r and the peak time t_p. For an over-damped system the peak time is not defined and the 10%-90% rise time t_{r1} is normally used.
- The closeness of the response $y(t)$ to the step input $r(t)$ is represented by the peak value or overshoot M_p and the settling time t_s. The settling time t_s is defined as the time required for the system to settle within a certain percentage e of the input amplitude.

Since these are contradictory requirements, a compromise must be obtained which leads to the problem of vector or multi-objective optimisation [11]. The favourable behaviour of the quality function with pure integral criterion is changed into an unfavourable quality landscape, especially through the insertion of ridges [5],[9],[12].

From integral and penalty terms to subdivided difference areas

An often used fitness function is shown below.

$$I = \int_0^T t^n \left(e^2(t) + a \cdot u^2(t)\right) dt \quad + W_1 \cdot t_r^{\,2} \quad + W_2 \cdot \left(M_p - M_p^{\,*}\right)^2 \Big|_{M_p > M_p^{\,*}} \tag{2}$$

In order to attribute the parts of integral to several characteristic sections see following function.

$$I = t^n \int_0^{t_{Mu}} e^2(t) dt \quad + t^n \int_{t_{Mu}}^{t_{tr}} e^2(t) dt + W_1 \cdot t_r^{\,2} \quad + t^n \int_{t_{tr}}^{T} e^2(t) dt + W_2 \cdot \Delta M_p \quad + a \cdot t^n \int_0^T u^2(t) dt \tag{3}$$

1. The time stamp t_{Mu} gives the maximum time for which the output signal is under the start value $y(0)$. It is a measure for how long the undershoot goes. This point of signal characteristic is often neglected and sub-aggregated in the error integral.
2. The second part of (3) treats with the rise time t_r. The rise time is evaluated in the error integral of ITSE and as extra penalty term again. The ITSE part represents a unit of area and is supported with a penalty term in time unit. This is over-dimensioned and leads to unnecessary deformation of weighting proportions.
3. The third part of equation above gives the treatment on the one hand of error signal after time stamp of rise time to the end of the time horizon and on the other hand of the penalty term of pass over of maximal admissible overshoot. Here the same annotations apply as in point 2 in principal.
4. The valuation of correcting variable renders the technically aim only very softly and conservatively. In practice a maximal value of $u(t)$ is given and is defined by physical constraints of the hardware. A maximal correcting variable always guarantees the fastest system possible.

New approach with special time sections of the signal error

The problem of double valuation and weighting within the approach above leads to multi-modality and ridges in the fitness landscape. The weighting factors for the used characteristic values for a desired aim of signal characteristic are set more intuitively. This selection pressure can be different for near or far optimisation. A mixture or swap of the selection pressure's relationship is possible and often leads to a local minimum.

A straight forward valuation of an signal error has to divide the signal space into time sections within which the characteristic values are defined. The more characteristic values are considered the more sections have to be divided. In general four sections or characteristic values are sufficient. The following equation represents the new fitness function which meets the requirements above.

$$I_{PITSE} = t^n \left(\int_{t_{y(t)<0}}^{t_{y(t)\geq 0}} y^2(t)dt + \int_{t=0}^{t_{y(t)\geq 0.9r(t)}} \left(\begin{cases} 0.9r(t) - y(t) & ,y(t) \leq 0 \\ 0.9r(t) & ,y(t) > 0 \end{cases} \right) dt + \int_{t_{y(t)>0.9r(t)}}^{t=T} e^2(t)dt + \int_{t_{y(t)>Mp \cdot r(t)}}^{t_{y(t)\leq Mp \cdot r(t)}} \left(y(t) - M_p r(t) \right)^2 dt \right) \tag{4}$$

Figure 2 shows the definition of different sections clearly.

$$I_{PITSE} = t^n \begin{pmatrix} 2A_{Mp}\big|_{Mp \cdot r(t) < y(t)} \\ + A_e\big|_{0.9r(t) < y(t) \leq Mp \cdot r(t)} \\ + A_{tr}\big|_{0 < y(t) \leq 0.9r(t)} \\ + 2A_{Mu}\big|_{y(t) \leq 0} \end{pmatrix} + A_U\big|_{u(t) > U\max}$$

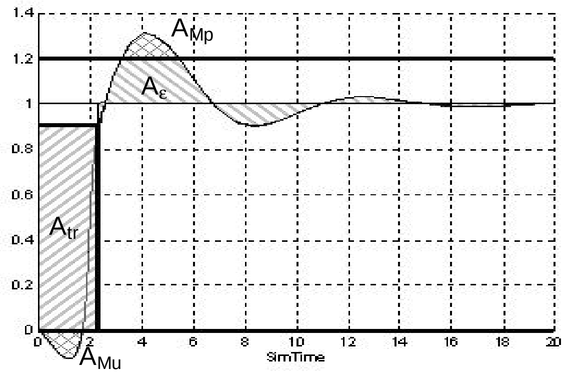

Figure 2: Piecewise Integral of Time weighted Squared Error - PITSE

The time sections rise time and epsilon area exclude each other. If the rise time gain of 0.9 percent of *r(t)* is reached, the rest of the time horizon greater than t_r is summarised by A_e no matter of gain value.

With embedding characteristic values in the integral form, a requirement transformation from value based to integral based representation prevents an non smooth fitness landscape and improve thus the convergence speed of ES. An implicitly good relationship between the characteristic values is build and an additional weighting is not necessary in general. If a special signal behaviour is required, the possibility of weighting each section is given too and should be used sparingly. The following enumeration describes each section of the Piecewise Integral of Time weighted Squared Error.

1. <u>Undershoot M_U:</u> Preventing an undershoot is often impossible but a minimising is required. This part of signal time horizon is treated specially because in optimisation with ES the start vector of the control parameter is often set to 1 and accordingly leads to an output signal which never lies

in a near surrounding of the input signal. In later generations this problem of far optimisation leads to near optimisation without changing the fitness function, a flow transition is given.

2. Rise time t_r: The rise time has a time unit and is transformed into a unit of area for the comparison with the other parts of the fitness function. The definition or existence of rise time, as above, depends on the user's view. A signal which never crosses the zero line of gain has a rise time defined not at all. For such signals the section of rise time in the new quality function is to extend in order to get a smooth transition from far to near optimisation. In our approach the rise time far from optimum is evaluated as the signal difference area between *r(t)* and *y(t)* from *t*=0 to *t* where the gain of *y(t)* is 0.9*r(t)*. This section of rise time includes also the undershoot area. The double valuation of undershoot leads to faster convergence by rising the selection pressure and prevents the hard switch of undershoot to rise time section. This is also the reason for setting the rise time starting point from 0.1 percent of *r(t)* to 0. The evaluation of error difference alone leads to a signal which minimises the error area, but it is not minimised in sense of signal length between *y(0)* to $y(t_r)$=0.9*r(t)*. The possibility of clinging of y(t) to 0.9r(t) but not beyond, often seen in non-linear systems, gives a small quality value and causes an unintentional signal behaviour. Such a peculiar bend of signals results from the direction where the signal comes from and can be traced back to different selection pressure in the fitness function.
3. Steady state error: The steady state error or epsilon surrounding is just the well known integral of squared error (ISE) and is described exhaustively in literature.
4. Overshoot M_p: For the overshoot or maximum peak M_p the same considerations as for undershoot are valid. This section is also double evaluated in the sense of more selection pressure and smooth fitness landscape.

The constricting by double evaluation of under- and overshoot in Figure 2 accelerates the far optimisation too. It is absolutely necessary to guarantee a steady selection pressure over the whole variety of output signals *y(t)*. If this is not possible, the EA finds the local gap and leads to lowest convergence speed or smooth to failing or to getting stuck in multi-modality [9].

Small non-linear example

The following non-linear example from the MATLAB Optimisation Toolbox in Figure 3 is used to illustrate the argumentation above. The limit is set to ±2 and the slew rate to ±0.8.

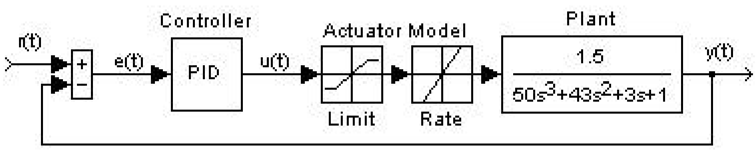

Figure 3: Non-linear example

The following simulations are run with the ES-CMA(1,8) [13] over 200 generations with start step size and start parameter of 1. Both fitness functions are simulated with the same start parameters for strategy and simulation environment, e.g. solver = ode23tb, simulation time = 50sec. In order to compare the best quality values, the best parameter vector for the controller of the old ITSE in (2) is finally calculated with the new PITSE. The characteristic values of rise time, overshoot and steady state error are shown to judge the quality of work of fitness functions. The new fitness function is simulated in all runs below as described in equation (4).

In Table 1 the mean value and the standard deviation of five optimisation runs are represented. The ITSE is adds with penalty terms of t_r and M_p with W_1 = W_2 = 1 as used in (2) left and compared with PITSE in (4) right. The optimal to reached characteristic values are set to M_p=1.2 and a minimal t_r.

Old Fitness Function (ITSE)					New Fitness Function (PITSE)				
Q	t_r	M_p	e_{ss}	Param [P, I, D]	Q	t_r	M_p	e_{ss}	Param [P, I, D]
93.645	7.544	1.033	0.027	3.18, 0.08, 13.20	30.956	7.191	1.070	<1e-3	3.79, 0.14, 17.76
0.134	0.004	<1e-3	<1e-3	<1e-2, <1e-2, 0.01	0.007	0.001	<1e-3	<1e-3	<1e-2, <1e-2,

Table 1: Old ITSE and new PITSE with signal requirements of M_p = 1.2 and t_r = minimal

For all optimisation runs in Table 1 the requirement for the correcting variable are met, and a optimum was reached at about 100 generations. Already these generally good requirements for t_r and M_p are not reached for the old ITSE and lead to an insufficient output behaviour. The higher quality value on the left in Table 1 is based on the longer rise time and on the not minimal steady state error up to the end of the simulation time. The aim is not reached and the weighting factors in (2) must be adjusted. A possible reason for the mismatch can be the conservative valuation of correcting variable *u(t)*.

In the second test, illustrated in Table 2, the characteristic values are set to M_p = 1 which means without an overshoot and a minimal t_r. The more the requirements are set to boundary constraints the more influence the additional penalty terms in (2) become and the more significant and difficult the set of right weightings is. To improve the selection pressure for both fitness functions only for the hard constraint M_p , W_2 is set to 10.

Old Fitness Function (ITSE)					New Fitness Function (PITSE)				
Q	t_r	M_p	e_{ss}	Param [P, I, D]	Q	t_r	M_p	e_{ss}	Param [P, I,
2.1987e	7.6081	0.0543	0.031	3.19, 0.08, 13.60	1.758e4	7.4658	0.0131	<1e-4	3.66, 0.13,
9.23318	1.6e-3	2.1e-4	<1e-4	1e-5, 1e-4, 6e-3	0.07914	3.3e-4	7.0e-5	<1e-4	5e-4, <1e-4,

Table 2: Old ITSE and new PITSE with signal requirements of M_p = 1 and t_r = minimal

In order to minimise the steady state error for the runs with the ITSE the time weighting exponent *n* is set to 4. Also this support does not leads to the desired aim of minimal error at the end of the simulation time. Figure 4 left shows both quality runs of mean values and standard deviations. The convergence courses of both integral criterions are equal nearly. But it is seen well in case of the PITSE a better fitness value is reached and is caused by a minimal overshoot and negligibly steady state error. In Figure 4 right, the output signal and the correcting variable of both fitness functions are presented. The closeness of *y(t)* to *r(t)* for the PITSE run can be traced back to a higher value of *u(t)* over the time. The constraint for M_p is reached and the smooth fitness landscape of the PITSE leads to a better optimum in fewer generations. In order to meet the requirement of overshoot for the run of the ITSE criterion the weight of M_p is to increase. This deform further the fitness landscape and can be the reason for getting stuck in a local optimum, seen in Figure 4.

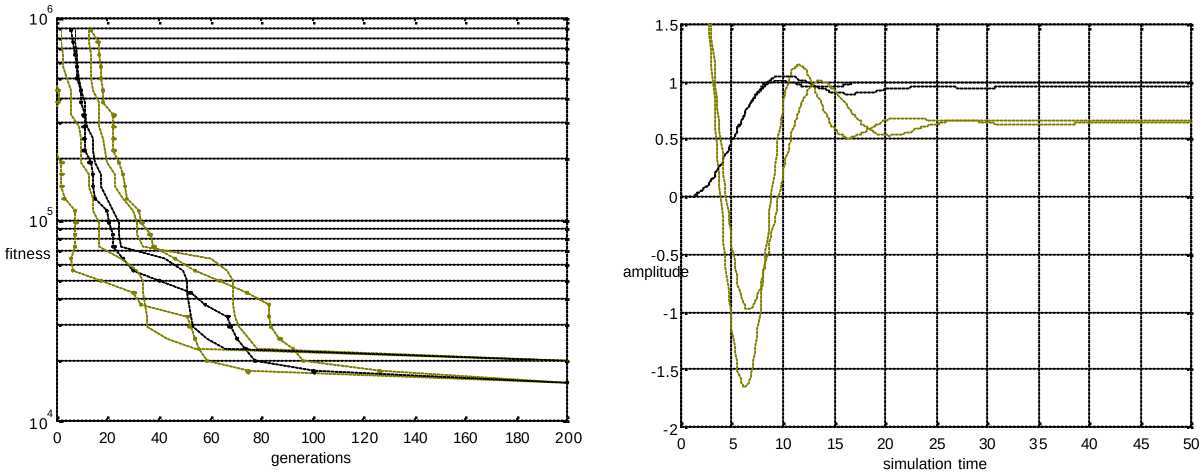

Figure 4: Left: Mean values and standard deviations of five runs in solid lines for ITSE and in solid lines with marker for PITSE. Right: *y(t)* and *u(t)* of best parameters in thin solid lines for ITSE and in fat solid lines for PITSE.

This example shows the significance of smooth fitness landscape already at problems with lower dimension.

Conclusions

For Evolutionary Algorithms the success of optimisation depends the more on the fitness function the more the algorithm tends to a strategy with step size control. The better the quality function describes the aim requirements in the sense of evolvable fitness landscape the better the optimum is found.

Constraint optimisation allows a definition of fitness for an algorithm in control parameter optimisation which is more realistic and easier to understand. But the constraint handling as an extra penalty term leads to the problem of double valuation of equal characteristics. An unlucky weighting relationship produces an non smooth fitness landscape and makes optimisation more difficult.

The new fitness function PITSE connects the error integral divided in time sections with the respective desire to influence a special characteristic value given by system requirements. One time section represents one characteristic value near the optimum. Far from the optimum the new fitness function owns inherently the attribute of smooth transition of judged signal characteristics. The advantage of an smooth fitness landscape leads to the improvement of optimisation with Evolutionary Algorithms.

Acknowledgements

This work was supported by the *Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie* under grant 01 IB 801.

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